Hungary-Israel Binational 1996

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1	Find all integer sequences of the form $x_i, 1 \leq i \leq 1997$, that satisfy $\sum_{k=1}^{1997} 2^{k-1} x_k^{1997} = 1996 \prod_{k=1}^{1997} x_k$.

2	$n > 2$ is an integer such that n^2 can be represented as a difference of cubes of 2 consecutive positive integers. Prove that n is a sum of 2 squares of positive integers, and that such n does exist.

3	A given convex polyhedron has no vertex which belongs to exactly 3 edges. Prove that the number of faces of the polyhedron that are triangles, is at least 8.

4	$a_1, a_2, \cdots, a_n$ is a sequence of real numbers, and $b_1, b_2, \cdots, b_n$ are real numbers that satisfy the condition $1 \geq b_1 \geq b_2 \geq \cdots \geq b_n \geq 0$. Prove that there exists a natural number $k \leq n$ that satisifes $|a_1 b_1 + a_2 b_2 + \cdots + a_n b_n| \leq |a_1 + a_2 + \cdots + a_k|$