Hungary-Israel Binational 1997

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Day 1

1	Is there an integer N such that $\left(\sqrt{1997}-\sqrt{1996}\right)^{1998} = \sqrt{N}-\sqrt{N-1}$?
2	Find all the real numbers α satisfy the following property: for any positive integer n there exists an integer m such that $\left\lvert \alpha - \frac{m}{n} \right\rvert < \frac{1}{3n}$.
3	Let ABC be an acute angled triangle whose circumcenter is O. The three diameters of the circumcircle that pass through A, B, and C, meet the opposite sides BC, CA, and AB at the points A_1, B_1 and C_1, respectively. The circumradius of ABC is of length $2P$, where P is a prime number. The lengths of OA_1, OB_1, OC_1 are integers. What are the lengths of the sides of the triangle?

Day 2

1	Determine the number of distinct sequences of letters of length 1997 which use each of the letters A, B, C (and no others) an odd number of times.
2	The three squares ACC_1A'', $ABB_1'A'$, $BCDE$ are constructed externally on the sides of a triangle ABC. Let P be the center of the square $BCDE$. Prove that the lines $A'C$, $A''B$, PA are concurrent.
3	Can a closed disk can be decomposed into a union of two congruent parts having no common point?