Hungary-Israel Binational 2000

www.artofproblemsolving.com/community/c3512

by N.T.TUAN, parmenides51

Day 1

1 Let S be the set of all partitions of 2000 (in a sum of positive integers). For every such partition p, we dene $f(p)$ to be the sum of the number of summands in p and the maximal summand in p. Compute the minimum of $f(p)$ when $p \in S.$

2 Prove or disprove: For any positive integer k there exists an integer $n > 1$ such that the binomial coeffcient $\binom{n}{i}$ is divisible by k for any $1 \leq i \leq n-1$.

3 Let ABC be a non-equilateral triangle. The incircle is tangent to the sides BC,CA,AB at A_1,B_1,C_1, respectively, and M is the orthocenter of triangle $A_1B_1C_1$. Prove that M lies on the line through the incenter and circumcenter of $\triangle ABC$.

Day 2

1 Let A and B be two subsets of $S = \{1,2,...,2000\}$ with $|A| \cdot |B| \geq 3999$. For a set X , let $X-X$ denotes the set $\{s-t | s,t \in X, s \not= t\}$. Prove that $(A-A) \cap (B-B)$ is nonempty.

2 For a given integer d, let us dene $S = \{m^2+dn^2 | m,n \in\mathbb{Z}\}$. Suppose that p,q are two elements of S , where p is prime and $p | q$. Prove that $r = q/p$ also belongs to S .

3 Let k and l be two given positive integers and $a_{ij}(1 \leq i \leq k, 1 \leq j \leq l)$ be kl positive integers. Show that if $q \geq p > 0$, then

$$(\sum_{j=1}^{l}(\sum_{i=1}^{k}a_{ij}^{p})^{q/p})^{1/q}\leq (\sum_{i=1}^{k}(\sum_{j=1}^{l}a_{ij}^{q})^{p/q})^{1/p}.$$