Hungary-Israel Binational 2005

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Day 1

1	Squares ABB_1A_2 and BCC_1B_2 are externally drawn on the hypotenuse AB and on the leg BC of a right triangle ABC . Show that the lines CA_2 and AB_2 meet on the perimeter of a square with the vertices on the perimeter of triangle ABC.
2	Let f be an increasing mapping from the family of subsets of a given nite set H into itself, i.e. such that for every $X \subseteq Y \subseteq H$ we have $f(X) \subseteq f(Y) \subseteq H$. Prove that there exists a subset H_0 of H such that $f(H_0) = H_0$.
3	Find all sequences $x_1, x_2, ..., x_n$ of distinct positive integers such that $\frac{1}{2} = \sum_{i=1}^{n} \frac{1}{x_i^2}$.

Day 2

1	Does there exist a sequence of 2005 consecutive positive integers that contains exactly 25 prime numbers?
2	Let F_n be the $n-$ th Fibonacci number (where $F_1 = F_2 = 1$). Consider the functions $f_n(x) = \| ... \|\ \|x\| - F_n\| - F_{n-1}\| - ... - F_2\| - F_1\|, g_n(x) = \|...\|\ x - 1\| - 1\| - ... - 1\|$ ($F_1 + ... + F_n$ ones). Show that $f_n(x) = g_n(x)$ for every real number x.
3	There are seven rods erected at the vertices of a regular heptagonal area. The top of each rod is connected to the top of its second neighbor by a straight piece of wire so that, looking from above, one sees each wire crossing exactly two others. Is it possible to set the respective heights of the rods in such a way that no four tops of the rods are coplanar and each wire passes one of the crossings from above and the other one from below?

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