Hungary-Israel Binational 2006

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by April

Day 1

1	If natural numbers x, y, p, n, k with $n > 1$ odd and p an odd prime satisfy $x^n + y^n = p^k$, prove that n is a power of p.
2	A block of size $a \times b \times c$ is composed of $1 \times 1 \times 2$ domino blocks. Assuming that each of the three possible directions of domino blocks occurs equally many times, what are the possible values of a, b, c?
3	Let $\mathcal{H} = A_1A_2 \dots A_n$ be a convex n-gon. For $i = 1, 2, \dots, n$, let A_i' be the point symmetric to A_i with respect to the midpoint of $A_{i-1}A_{i+1}$ (where $A_{n+1} = A_1$). We say that the vertex A_i is *good* if A_i' lies inside $\mathcal{H}$. Show that at least $n - 3$ vertices of $\mathcal{H}$ are *good*.

Day 2

1	A point P inside a circle is such that there are three chords of the same length passing through P. Prove that P is the center of the circle.
2	If x, y, z are nonnegative real numbers with the sum 1, find the maximum value of $S = x^2(y + z) + y^2(z + x) + z^2(x + y)$ and $C = x^2y + y^2z + z^2x$.
3	A group of 100 students numbered 1 through 100 are playing the following game. The judge writes the numbers 1, 2, $\dots$, 100 on 100 cards, places them on the table in an arbitrary order and turns them over. The students 1 to 100 enter the room one by one, and each of them flips 50 of the cards. If among the cards flipped by student j there is card j, he gains one point. The flipped cards are then turned over again. The students cannot communicate during the game nor can they see the cards flipped by other students. The group wins the game if each student gains a point. Is there a strategy giving the group more than 1 percent of chance to win?