Hungary-Israel Binational 2008

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by bambaman, freemind

Day 1

1   Find the largest value of n, such that there exists a polygon with n sides, 2 adjacent sides of length 1, and all his diagonals have an integer length.

2   For every natural number t, $f(t)$ is the probability that if a fair coin is tossed t times, the number of times we get heads is 2008 more than the number of tails. What is the value of t for which $f(t)$ attains its maximum? (if there is more than one, describe all of them)

3   A rectangle D is partitioned in several (≥ 2) rectangles with sides parallel to those of D. Given that any line parallel to one of the sides of D, and having common points with the interior of D, also has common interior points with the interior of at least one rectangle of the partition; prove that there is at least one rectangle of the partition having no common points with D's boundary.

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Day 2

1   Prove that: $\sum_{i=1}^{n^2} \lfloor \frac{i}{3} \rfloor = \frac{n^2(n^2-1)}{6}$
For all $n \in N$.

2   The sequence a_n is defined as follows: $a_0 = 1, a_1 = 1, a_{n+1} = \frac{1+a_n^2}{a_{n-1}}$.
Prove that all the terms of the sequence are integers.

3   P and Q are 2 points in the area bounded by 2 rays, e and f, coming out from a point O. Describe how to construct, with a ruler and a compass only, an isosceles triangle ABC, such that his base AB is on the ray e, the point C is on the ray f, P is on AC, and Q on BC.