Bulgaria Team Selection Test 2003

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by augustin_p, subham1729

–	Day 1
1	Cut 2003 disjoint rectangles from an acute-angled triangle ABC, such that any of them has a parallel side to AB and the sum of their areas is maximal.
2	Find all $f : R - R$ such that $f(x^2 + y + f(y)) = 2y + f(x)^2$
3	Some of the vertices of a convex n-gon are connected by segments, such that any two of them have no common interior point. Prove that, for any n points in general position, there exists a one-to-one correspondence between the points and the vertices of the n gon, such that any two segments between the points, corresponding to the respective segments from the n gon, have no common interior point.
–	Day 2
4	Is it true that for any permulation $a_1, a_2....., a_{2002}$ of $1, 2...., 2002$ there are positive integers m, n of the same parity such that $0 < m < n < 2003$ and $a_m + a_n = 2a_{\frac{m+n}{2}}$
5	Let $ABCD$ be a circumscribed quadrilateral and let P be the orthogonal projection of its in center on AC. Prove that $\angle APB = \angle APD$
6	In natural numbers m, n Solve : $n(n+1)(n+2)(n+3) = m(m+1)^2(m+2)^3(m+3)^4$