Bulgaria Team Selection Test 2005

www.artofproblemsolving.com/community/c3523

by Mladenov

Day 1

1 Let ABC be an acute triangle. Find the locus of the points M, in the interior of $\triangle ABC$, such that $AB-FG=\frac{MF.AG+MG.BF}{CM}$, where F and G are the feet of the perpendiculars from M to the lines BC and AC, respectively.

2 Find the number of the subsets B of the set $\{1,2,\cdots,2005\}$ such that the sum of the elements of B is congruent to 2006 modulo 2048

3 Let $\mathbb{R}^*$ be the set of non-zero real numbers. Find all functions $f:\mathbb{R}^*\to\mathbb{R}^*$ such that $f(x^2+y)=(f(x))^2+\frac{f(xy)}{f(x)}$, for all $x,y\in\mathbb{R}^*$ and $-x^2\neq y$.

Day 2

4 Let a_i and b_i, where $i\in\{1,2,\dots,2005\}$, be real numbers such that the inequality $(a_ix-b_i)^2\geq\sum_{j=1,j\neq i}^{2005}(a_jx-b_j)$ holds for all $x\in\mathbb{R}$ and all $i\in\{1,2,\dots,2005\}$. Find the maximum possible number of positive numbers amongst a_i and b_i, $i\in\{1,2,\dots,2005\}$.

5 Let ABC, $AC\neq BC$, be an acute triangle with orthocenter H and incenter I. The lines CH and CI meet the circumcircle of $\triangle ABC$ at points D and L, respectively. Prove that $\angle CIH=90^\circ$ if and only if $\angle IDL=90^\circ$

6 In a group of nine persons it is not possible to choose four persons such that every one knows the three others. Prove that this group of nine persons can be partitioned into four groups such that nobody knows anyone from his or her group.