Final Round - Korea 1993

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Day 1

1 Consider a 9×9 array of white squares. Find the largest $n \in \mathbb{N}$ with the property: No matter how one chooses n out of 81 white squares and color in black, there always remains a 1×4 array of white squares (either vertical or horizontal).

2 Let be given a triangle ABC with $BC = a, CA = b, AB = c$. Find point P in the plane for which $aAP^2 + bBP^2 + cCP^2$ is minimum, and compute this minimum.

3 Find the smallest $x \in \mathbb{N}$ for which $\frac{7x^{25}-10}{83}$ is an integer.

Day 2

4 An integer which is the area of a right-angled triangle with integer sides is called *Pythagorean*. Prove that for every positive integer $n > 12$ there exists a Pythagorean number between n and $2n$.

5 Given $n \in \mathbb{N}$, nd all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x \in \mathbb{R}$,

$$\sum_{k=0}^{n} \binom{n}{k} f(x^{2^k}) = 0.$$

6 Consider a triangle ABC with $BC = a, CA = b, AB = c$. Let D be the midpoint of BC and E be the intersection of the bisector of A with BC . The circle through A, D, E meets AC, AB again at F, G respectively. Let $H \neq B$ be a point on AB with $BG = GH$. Prove that triangles EBH and ABC are similar and nd the ratio of their areas.