**Final Round - Korea 1998**

www.artofproblemsolving.com/community/c3534

by Peter, ComplexPhi, bah_luckyboy, chien than

**Day 1**

**1** Find all pairwise relatively prime positive integers $l, m, n$ such that

$$(l+m+n)\left(\frac{1}{l}+\frac{1}{m}+\frac{1}{n}\right)$$

is an integer.

**2** Let $D$,$E$,$F$ be points on the sides $BC$,$CA$,$AB$ respectively of a triangle $ABC$. Lines $AD$,$BE$,$CF$ intersect the circumcircle of $ABC$ again at $P$,$Q$,$R$, respectively.Show that:

$$\frac{AD}{PD}+\frac{BE}{QE}+\frac{CF}{RF}\geq 9$$

and find the cases of equality.

**3** Denote by $\phi(n)$ for all $n \in \mathbb{N}$ the number of positive integer smaller than $n$ and relatively prime to $n$. Also, denote by $\omega(n)$ for all $n \in \mathbb{N}$ the number of prime divisors of $n$. Given that $\phi(n)|n-1$ and $\omega(n) \leq 3$. Prove that $n$ is a prime number.

**Day 2**

**1** Let $x, y, z$ be positive real numbers satisfying $x+y+z=xyz$. Prove that:

$$\frac{1}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+y^2}}+\frac{1}{\sqrt{1+z^2}}\leq \frac{3}{2}$$

**2** Let $I$ be the incenter of triangle $ABC$, $O_1$ a circle through $B$ tangent to $CI$, and $O_2$ a circle through $C$ tangent to $BI$. Prove that $O_1$,$O_2$ and the circumcircle of $ABC$ have a common point.

**3** Let $F_n$ be the set of all bijective functions $f:\{1,2,\ldots,n\}\rightarrow\{1,2,\ldots,n\}$ that satisfy the conditions:

1. $f(k)\leq k+1$ for all $k\in\{1,2,\ldots,n\}$
2. $f(k)\neq k$ for all $k\in\{2,3,\ldots,n\}$

Find the probability that $f(1)\neq 1$ for $f$ randomly chosen from $F_n$.