Final Round - Korea 2009

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Day 1

1 a, b, c are the length of three sides of a triangle. Let $A = \frac{a^2+bc}{b+c} + \frac{b^2+ca}{c+a} + \frac{c^2+ab}{a+b}$, $B = \frac{1}{\sqrt{(a+b-c)(b+c-a)}} + \frac{1}{\sqrt{(b+c-a)(c+a-b)}} + \frac{1}{\sqrt{(c+a-b)(a+b-c)}}$. Prove that $AB \geq 9$.

2 ABC is an obtuse triangle. (angle B is obtuse) Its circumcircle is O. A tangent line for O passing C meets with AB at B_1. Let O_1 be a circumcenter of triangle AB_1C. B_2 is a point on the segment BB_1. Let C_1 be a contact point of the tangent line for O passing B_2, which is more closer to C. Let O_2 be a circumcenter of triangle AB_2C_1. Prove that if OO_2 and AO_1 is perpendicular, then five points O, O_2, O_1, C_1, C are concyclic.

3 2008 white stones and 1 black stone are in a row. An 'action' means the following: select one black stone and change the color of neighboring stone(s).
Find all possible initial position of the black stone, to make all stones black by finite actions.

Day 2

4 ABC is an acute triangle. (angle C is bigger than angle B) Let O be a center of the circle which passes B and tangents to AC at C. O meets the segment AB at D. CO meets the circle (O) again at P, a line, which passes P and parallel to AO, meets AC at E, and EB meets the circle (O) again at L. A perpendicular bisector of BD meets AC at F and LF meets CD at K. Prove that two lines EK and CL are parallel.

5 There is a $m \times (m-1)$ board. (i.e. there are $m+1$ horizontal lines and m vertical lines) A stone is put on an intersection of the lowest horizontal line. Now two players move this stone with the following rules.
(i) Each players move the stone to a neighboring intersection along a segment, by turns.
(ii) A segment, which is already passed by the stone, cannot be used more.
(iii) One who cannot move the stone anymore loses.
Prove that there is a winning strategy for the former player.

6 Find all pairs of two positive integers (m, n) satisfying $3^m - 7^n = 2$.