

**Croatia Team Selection Test 2006**

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- 1 Find all natural numbers that can be expressed in a unique way as a sum of ve or less perfect squares.

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- 2 Assume that  $a, b,$  and  $c$  are positive real numbers for which  $(a + b)(a + c)(b + c) = 1$ . Prove that  $ab + bc + ca \leq \frac{3}{4}$ .

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- 3 Let  $ABC$  be a triangle for which  $AB + BC = 3AC$ . Let  $D$  and  $E$  be the points of tangency of the incircle with the sides  $AB$  and  $BC$  respectively, and let  $K$  and  $L$  be the other endpoints of the diameters originating from  $D$  and  $E$ . Prove that  $C, A, L,$  and  $K$  lie on a circle.

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- 4 Find all natural solutions of  $3^x = 2^x y + 1$ .

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