Croatia Team Selection Test 2008

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by April, easternlatincup

1 Let x, y, z be positive numbers. Find the minimum value of: $(a)\quad \frac{x^2+y^2+z^2}{xy+yz}$ $(b)\quad \frac{x^2+y^2+2z^2}{xy+yz}$

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2 For which $n\in\mathbb{N}$ do there exist rational numbers a,b which are not integers such that both $a+b$ and a^n+b^n are integers?

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3 Point M is taken on side BC of a triangle ABC such that the centroid T_c of triangle ABM lies on the circumcircle of $\triangle ACM$ and the centroid T_b of $\triangle ACM$ lies on the circumcircle of $\triangle ABM$. Prove that the medians of the triangles ABM and ACM from M are of the same length.

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4 Let S be the set of all odd positive integers less than $30m$ which are not multiples of 5, where m is a given positive integer. Find the smallest positive integer k such that each k-element subset of S contains two distinct numbers, one of which divides the other.
