Croatia Team Selection Test 2009

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–	MEMO TST 1
1	Prove for all positive reals a,b,c,d: $\frac{a-b}{b+c}+\frac{b-c}{c+d}+\frac{c-d}{d+a}+\frac{d-a}{a+b}\geq 0$
2	On sport games there was 1991 participant from which every participant knows at least n other participants(friendship is mutual). Determine the lowest possible n for which we can be sure that there are 6 participants between which any two participants know each other.
3	On sides AB and AC of triangle ABC there are given points D, E such that DE is tangent of circle inscribed in triangle ABC and $DE \parallel BC$. Prove $AB+BC+CA\geq 8DE$
4	Prove that there are infinite many positive integers n such that $n^2+1\mid n!$, and infinite many of those for which $n^2+1\nmid n!$.
–	MEMO TST 2
1	Determine the lowest positive integer n such that following statement is true: If polynomial with integer coefficients gets value 2 for n different integers, then it can't take value 4 for any integer.
2	In each field of 2009*2009 table you can write either 1 or -1. Denote Ak multiple of all numbers in k-th row and Bj the multiple of all numbers in j-th column. Is it possible to write the numbers in such a way that $\sum_{i=1}^{2009}Ai+\sum_{i=1}^{2009}Bi=0$?
3	It is given a convex quadrilateral $ABCD$ in which $\angle B+\angle C<180^0$. Lines AB and CD intersect in point E. Prove that $CD*CE=AC^2+AB*AE \leftrightarrow \angle B=\angle D$
4	Determine all triplets off positive integers (a,b,c) for which $\mid 2^a-b^c\mid=1$
–	IMO TST
1	Solve in the set of real numbers: $$3\left(x^2+y^2+z^2\right)=1,$$ $$x^2y^2+y^2z^2+z^2x^2=xyz\left(x+y+z\right)^3.$$

2 Every natural number is coloured in one of the k colors. Prove that there exist four distinct natural numbers a, b, c, d, all coloured in the same colour, such that $ad = bc$, $\frac{b}{a}$ is power of 2 and $\frac{c}{a}$ is power of 3.

3 A triangle ABC is given with $|AB| > |AC|$. Line l tangents in a point A the circumcirle of ABC. A circle centered in A with radius $|AC|$ cuts AB in the point D and the line l in points E, F (such that C and E are in the same halfplane with respect to AB). Prove that the line DE passes through the incenter of ABC.

4 Determine all natural n for which there exists natural m divisible by all natural numbers from 1 to n but not divisible by any of the numbers $n+1$, $n+2$, $n+3$.