IMS 2006

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1 Prove that for each $m\geq 1$:

$$\sum_{|k|<\sqrt m}\binom{2m}{m+k}\geq 2^{2m-1}$$

Maybe probabilistic method works

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2 For each subset C of $\mathbb N$, Suppose $C\oplus C=\{x+y|x,y\in C, x\neq y\}$. Prove that there exist a unique partition of $\mathbb N$ to sets A, B that $A\oplus A$ and $B\oplus B$ do not have any prime numbers.

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3 G is a group that order of each element of it Commutator group is finite. Prove that subset of all elemets of G which have finite order is a subgroup og G.

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4 Assume that X is a seperable metric space. Prove that if $f:X\longrightarrow\mathbb R$ is a function that $\lim_{x\to a}f(x)$ exists for each $a\in\mathbb R$. Prove that set of points in which f is not continuous is countable.

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5 Suppose that $a_1,a_2,\dots,a_k\in\mathbb C$ that for each $1\leq i\leq k$ we know that $|a_k|=1$. Suppose that

$$\lim_{n\to\infty}\sum_{i=1}^k a_i^n=c.$$

Prove that $c=k$ and $a_i=1$ for each i.
