Macedonia National Olympiad 2000

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by WakeUp

1 Let AB be a diameter of a circle with centre O, and CD be a chord perpendicular to AB. A chord AE intersects CO at M, while DE and BC intersect at N. Prove that $CM:CO=CN:CB$.

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2 If $a_1,a_2,a_3\dots a_n$ are positive numbers, find the maximum value of

$$\frac{a_1a_2\dots a_{n-1}a_n}{(1+a_1)(a_1+a_2)\dots(a_{n-1}+a_n)(a_n+2^{n+1})}$$

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3 In a triangle with sides a,b,c,t_a,t_b,t_c are the corresponding medians and D the diameter of the circumcircle. Prove that

$$\frac{a^2+b^2}{t_c}+\frac{b^2+c^2}{t_a}+\frac{c^2+a^2}{t_b}\le 6D$$

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4 Let a,b be coprime positive integers. Show that the number of positive integers n for which the equation $ax+by=n$ has no positive integer solutions is equal to $\frac{(a-1)(b-1)}{2}-1$.
