Macedonia National Olympiad 2010

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by StefanS

1 Solve the equation

$$x^3 + 2y^3 - 4x - 5y + z^2 = 2012,$$

in the set of integers.

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2 Let a, b, c be positive real numbers for which $a + b + c = 3$. Prove the inequality

$$\frac{a^3+2}{b+2} + \frac{b^3+2}{c+2} + \frac{c^3+2}{a+2} \geq 3$$

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3 A total of 2010 coins are distributed in 5 boxes. At the beginning the quantities of coins in the boxes are consecutive natural numbers. Martha should choose and take one of the boxes, but before that she can do the following transformation finitely many times: from a box with at least 4 coins she can transfer one coin to each of the other boxes.
What is the maximum number of coins that Martha can take away?

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4 The point O is the centre of the circumscribed circle of the acute-angled triangle ABC. The line AO cuts the side BC in point N, and the line BO cuts the side AC at point M. Prove that if $CM = CN$, then $AC = BC$.

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5 Let the boxes in picture 1 be marked as in picture 2 below (from top to bottom in layers). In one move it is allowed to switch the empty box with another box adjacent to it (two boxes are adjacent if they share a common side). Can the arrangement of the numbers in picture 3 be obtained after finitely many moves?
