Macedonia National Olympiad 2013

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1 Let p,q,r be prime numbers. Solve the equation $p^{2q}+q^{2p}=r$

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2 2^n coins are given to a couple of kids. Interchange of the coins occurs when some of the kids has at least half of all the coins. Then from the coins of one of those kids to the all other kids are given that much coins as the kid already had. In case when all the coins are at one kid there is no possibility for interchange. What is the greatest possible number of consecutive interchanges? (n is natural number)

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3 Acute angle triangle is given such that BC is the longest side. Let E and G be the intersection points from the altitude from A to BC with the circumscribed circle of triangle ABC and BC respectively. Let the center O of this circle is positioned on the perpendicular line from A to BE. Let EM be perpendicular to AC and EF be perpendicular to AB. Prove that the area of $FBEG$ is greater than the area of MFE.

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4 Let a,b,c be positive real numbers such that $a^4+b^4+c^4=3$. Prove that

$$\frac{9}{a^2+b^4+c^6}+\frac{9}{a^4+b^6+c^2}+\frac{9}{a^6+b^2+c^4}\leq a^6+b^6+c^6+6$$

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5 An arbitrary triangle ABC is given. There are 2 lines, p and q, that are not parallel to each other and they are not perpendicular to the sides of the triangle. The perpendicular lines through points A, B and C to line p we denote with p_a,p_b,p_c and the perpendicular lines to line q we denote with q_a,q_b,q_c. Let the intersection points of the lines p_a,q_a,p_b,q_b,p_c and q_c with q_b,p_b,q_c,p_c,q_a and p_a are K,L,P,Q,N and M. Prove that KL, MN and PQ intersect in one point.
