Macedonia National Olympiad 2014
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1 In a plane, 2014 lines are distributed in 3 groups. in every group all the lines are parallel between themselves. What is the maximum number of triangles that can be formed, such that every side of such triangle lie on one of the lines?

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2 Solve the following equation in $\mathbb{Z}$:

$$3^{2a+1}b^2+1=2^c$$

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3 Let k_1, k_2 and k_3 be three circles with centers O_1, O_2 and O_3 respectively, such that no center is inside of the other two circles. Circles k_1 and k_2 intersect at A and P, circles k_1 and k_3 intersect and C and P, circles k_2 and k_3 intersect at B and P. Let X be a point on k_1 such that the line XA intersects k_2 at Y and the line XC intersects k_3 at Z, such that Y is nor inside k_1 nor inside k_3 and Z is nor inside k_1 nor inside k_2.

a) Prove that $\triangle XYZ$ is simular to $\triangle O_1O_2O_3$
b) Prove that the $P_{\triangle XYZ} \leq 4P_{\triangle O_1O_2O_3}$. Is it possible to reach equation?

Note: P denotes the area of a triangle.

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4 Let a, b, c be real numbers such that $a+b+c=4$ and $a, b, c > 1$. Prove that:

$$\frac{1}{a-1}+\frac{1}{b-1}+\frac{1}{c-1} \geq \frac{8}{a+b}+\frac{8}{b+c}+\frac{8}{c+a}$$

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5 From an equilateral triangle with side 2014 we cut off another equilateral triangle with side 214, such that both triangles have one common vertex and two of the side of the smaller triangles lie on two of the side of the bigger triangle. Is it possible to cover this figure with figures in the picture without overlapping (rotation is allowed) if all figures are made of equilateral triangles with side 1? Explain the answer!