Belarusian National Olympiad 2008

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– Day 1

– Prove, if $x^2+y=y^2+z=z^2+x$, then
$$x^3+y^3+z^3=xy^2+yz^2+zx^2$$

– $ABCD$ - quadrilateral inscribed in circle, and $AB=BC, AD=3DC$. Point R is on the BD and $DR=2RB$. Point Q is on AR and $\angle ADQ=\angle BDQ$. Also $\angle ABQ+\angle CBD=\angle QBD$. AB intersect line DQ in point P.
Find $\angle APD$

– a)Find one natural k , for which exist natural a,b,c and
$$a^2+k^2=b^2+(k+1)^2=c^2+(k+2)^2(*)$$
b)Prove, that there are infinitely many such k, for which condition (*) is true.
c)Prove, that if for some k there are such a,b,c that condition (*) is true, than $144|abc$
d)Prove, that there are not such natural k for which exist natural a,b,c,d and
$$a^2+k^2=b^2+(k+1)^2=c^2+(k+2)^2=d^2+(k+3)^2$$

– Find maximal numbers of planes, such there are 6 points and
1) 4 or more points lies on every plane.
2) No one line passes through 4 points.

– Day 2

– For $x_1,x_2,...,x_n \geq 0$ prove
$$\frac{x_1(2x_1-x_2-x_3)}{x_2+x_3}+\frac{x_2(2x_2-x_3-x_4)}{x_3+x_4}+...+\frac{x_n(2x_n-x_1-x_2)}{x_1+x_2} \geq 0$$

– Point O - center of circle ω. Point A is outside ω. Secant goes through A and intersect circle in points X and Y. Point X' is symmetric for point X with respect to line OA.
Prove, that point of intersection of OA and $X'Y$ is independent from the choice of secant.

– a, b, c - are sides of triangle T. It is known, that if we increase any one side by 1, we get new
a) triangle
b)acute triangle

Find minimal possible area of triangle T in case of a) and in case b)

– Find all pairs (n, m) $m \geq n \geq 3$, for which exist such table $n \times m$ with numbers in every cell, and sum of numbers in every 2×2 is negative and sum of numbers in every 3×3 is positive.
