Serbia Team Selection Test 2012

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by Djile

–	TST
–	Additional TST
1	Let $P(x)$ be a polynomial of degree 2012 with real coefficients satisfying the condition $$P(a)^3+P(b)^3+P(c)^3 \geq 3P(a)P(b)P(c),$$ for all real numbers a,b,c such that $a+b+c=0$. Is it possible for $P(x)$ to have exactly 2012 distinct real roots?
2	Let $\sigma(x)$ denote the sum of divisors of natural number x, including 1 and x. For every $n\in \mathbb{N}$ define $f(n)$ as number of natural numbers $m, m\leq n$, for which $\sigma(m)$ is odd number. Prove that there are infinitely many natural numbers n, such that $f(n)\|n$.
3	Let P and Q be points inside triangle ABC satisfying $\angle PAC = \angle QAB$ and $\angle PBC = \angle QBA$. a) Prove that feet of perpendiculars from P and Q on the sides of triangle ABC are concyclic. b) Let D and E be feet of perpendiculars from P on the lines BC and AC and F foot of perpendicular from Q on AB. Let M be intersection point of DE and AB. Prove that $MP\perp CF$.