**National Science Olympiad 2002**

www.artofproblemsolving.com/community/c3645

by chaotic_iak

**1** Prove that $n^4 - n^2$ is divisible by $12$ for all integers $n > 1$.

**2** Five dice are rolled. The product of the faces are then computed. Which result has a larger probability of occurring; $180$ or $144$?

**3** Find all solutions (real and complex) for $x, y, z$, given that:

$$x + y + z = 6x^2 + y^2 + z^2 = 12x^3 + y^3 + z^3 = 24$$

**4** Given a triangle $ABC$ where $AC > BC$, $D$ is located on the circumcircle of $ABC$ such that $D$ is the midpoint of the arc $AB$ that contains $C$. $E$ is a point on $AC$ such that $DE$ is perpendicular to $AC$. Prove that $AE = EC + CB$.

**5** Nine of the numbers $4, 5, 6, 7, 8, 12, 13, 16, 18, 19$ are going to be inputted to the empty cells in the following table:

| | | |
|---|---|---|
| 10 | | |
| | | 9 |
| | 3 | |
| 11 | | 17 |
| | 20 | |

such that each row sums to the same number, and each column sums to the same number. Determine all possible arrangements.

**6** Find all primes $p$ such that $4p^2 + 1$ and $6p^2 + 1$ are both primes.

**7** Let $ABCD$ be a rhombus where $\angle DAB = 60^\circ$, and $P$ be the intersection between $AC$ and $BD$. Let $Q, R, S$ be three points on the boundary of $ABCD$ such that $PQRS$ is a rhombus. Prove that exactly one of $Q, R, S$ lies on one of $A, B, C, D$.