

National Science Olympiad 2004

www.artofproblemsolving.com/community/c3647

by wangsac1

Day 1

1 Determine the number of positive odd and even factor of $5^6 - 1$.

2 When filled with an cold water using a particular cold water tap, a tank will be full in 14 minutes. In 21 minutes, the full tank could be emptied by opening a hole on the base of the tank. If the cold water tap and the hot water tap are opened simultaneously (allowing hot and cold water fill the tank), and the hole on the base of the tank is opened, the tank will be full in 12.6 minutes. Determine the number of minutes needed to fill the tank with hot water until the tank is full, assuming at first the tank is empty and the hole is closed.

3 In how many ways can we change the sign $*$ with $+$ or $-$, such that the following equation is true?

$$1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10 = 29$$

4 There exists 4 circles, a, b, c, d , such that a is tangent to both b and d , b is tangent to both a and c , c is both tangent to b and d , and d is both tangent to a and c . Show that all these tangent points are located on a circle.

Day 2

1

$$a + 4b + 9c + 16d + 25e + 36f + 49g = 1$$

$$4a + 9b + 16c + 25d + 36e + 49f + 64g = 12$$

$$9a + 16b + 25c + 36d + 49e + 64f + 81g = 123$$

Determine the value of $16a + 25b + 36c + 49d + 64e + 81f + 100g$.

2 Quadratic equation $x^2 + ax + b + 1 = 0$ have 2 positive integer roots, for integers a, b . Show that $a^2 + b^2$ is not a prime.

3 Given triangle ABC with C a right angle, show that the diameter of the incenter is $a + b - c$, where $a = BC$, $b = CA$, and $c = AB$.

- 4 8. Sebuah lantai luasnya 3 meter persegi ditutupi lima buah karpet dengan ukuran masing-masing 1 meter persegi. Buktikan bahwa ada dua karpet yang tumpang tindih dengan luas tumpang tindih minimal 0,2 meter persegi.

A floor of a certain room has a $3 m^2$ area. Then the floor is covered by 5 rugs, each has an area of $1 m^2$. Prove that there exists 2 overlapping rugs, with at least $0.2 m^2$ covered by both rugs.
