Azerbaijan BMO TST 2016

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–	TST 1	

1 A line is called $good$ if it bisects perimeter and area of a figure at the same time.Prove that:

a) all of the good lines in a triangle concur.
b) all of the good lines in a regular polygon concur too.

2 Set A consists of natural numbers such that these numbers can be expressed as $2x^2+3y^2$, where x and y are integers. $(x^2+y^2\neq 0)$ $a)$ Prove that there is no perfect square in the set A. $b)$ Prove that multiple of odd number of elements of the set A cannot be a perfect square.

3 k is a positive integer. A company has a special method to sell clocks. Every customer can reason with two customers after he has bought a clock himself $;$ it's not allowed to reason with an agreed person. These new customers can reason with other two persons and it goes like this.. If both of the customers agreed by a person could play a role (it can be directly or not) in buying clocks by at least k customers, this person gets a present. Prove that, if n persons have bought clocks, then at most $\frac{n}{k+2}$ presents have been accepted.

4 Find all functions $f:\mathbb{N}\to\mathbb{N}$ such that

$$f(f(n))=n+2015$$

where $n\in\mathbb{N}$.

– TST 3

1 Find all n natural numbers such that for each of them there exist p,q primes such that these terms satisfy.

$1.$ $p+2=q$ $2.$ 2^n+p and 2^n+q are primes.

2 n triangle ABC the bisector of $\angle BAC$ intersects the side BC at the point D.The circle ω passes through A and tangent to the side BC at D.AC and ω intersects at M second time , BM and ω intersects at P second time. Prove that point P lies on median of triangle ABD.

3 There are some checkers in $n\cdot n$ size chess board.Known that for all numbers $1\le i,j\le n$ if checkwork in the intersection of i th row and j th column is empty,so the number of checkers that are in this row and column is at least n.Prove that there are at least $\frac{n^2}{2}$ checkers in chess board.

4 For all numbers $n \geq 1$ does there exist infinite positive numbers sequence $x_1, x_2, ..., x_n$ such that $x_{n+2} = \sqrt{x_{n+1}} - \sqrt{x_n}$

– TST 4

1 Let a, b, c be nonnegative real numbers.Prove that $3(a^2 + b^2 + c^2) \geq (a + b + c)(\sqrt{ab} + \sqrt{bc} + \sqrt{ca}) + (a - b)^2 + (b - c)^2 + (c - a)^2 \geq (a + b + c)^2$.

2 There are 100 students who praticipate at exam.Also there are 25 members of jury.Each student is checked by one jury.Known that every student likes 10 jury $a)$ Prove that we can select 7 jury such that any student likes at least one jury. $b)$ Prove that we can make this every student will be checked by the jury that he likes and every jury will check at most 10 students.

3 a, b are positive integers and $(a! + b!)|a!b!$.Prove that $3a \geq 2b + 2$.

4 Let ABC be an acute triangle and let M be the midpoint of AC. A circle ω passing through B and M meets the sides AB and BC at points P and Q respectively. Let T be the point such that $BPTQ$ is a parallelogram. Suppose that T lies on the circumcircle of ABC. Determine all possible values of $\frac{BT}{BM}$.

