**Bosnia Herzegovina Team Selection Test 2007**

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**Day 1**

**1** Let $ABC$ be a triangle such that length of internal angle bisector from $B$ is equal to $s$. Also, length of external angle bisector from $B$ is equal to $s_1$. Find area of triangle $ABC$ if $\frac{AB}{BC} = k$

**2** Find all pairs of integers $(x, y)$ such that $x(x+2) = y^2(y^2+1)$

**3** Find all $x \in \mathbb{Z}$ and $a \in \mathbb{R}$ satisfying

$$\sqrt{x^2-4} + \sqrt{x+2} = \sqrt{x-a} + a$$

**Day 2**

**4** Let $P(x)$ be a polynomial such that $P(x) = x^3 - 2x^2 + bx + c$. Roots of $P(x)$ belong to interval $(0, 1)$. Prove that $8b + 9c \leq 8$. When does equality hold?

**5** Triangle $ABC$ is right angled such that $\angle ACB = 90^\circ$ and $\frac{AC}{BC} = 2$. Let the line parallel to side $AC$ intersects line segments $AB$ and $BC$ in $M$ and $N$ such that $\frac{CN}{BN} = 2$. Let $O$ be the intersection point of lines $CM$ and $AN$. On segment $ON$ lies point $K$ such that $OM + OK = KN$. Let $T$ be the intersection point of angle bisector of $\angle ABC$ and line from $K$ perpendicular to $AN$. Determine value of $\angle MTB$.

**6** The set $A$ has exactly $n > 4$ elements. Ann chooses $n+1$ distinct subsets of $A$, such that every subset has exactly $3$ elements. Prove that there exist two subsets chosen by Ann which have exactly one common element.