Bosnia Herzegovina Team Selection Test 2008

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Day 1

1 Prove that in an isosceles triangle $\triangle ABC$ with $AC = BC = b$ following inequality holds $b > \pi r$, where r is inradius.

2 Find all pairs of positive integers m and n that satisfy (both) following conditions:

(i) $m^2 - n$ divides $m + n^2$

(ii) $n^2 - m$ divides $n + m^2$

3 30 persons are sitting at round table. $30 - N$ of them always speak true (”true speakers”) while the other N of them sometimes speak true sometimes not (”lie speakers”). Question: ”Who is your right neighbour - ”true speaker” or ”lie speaker” ?” is asked to all 30 persons and 30 answers are collected. What is maximal number N for which (with knowledge of these answers) we can always be sure (decide) about at least one person who is ”true speaker”.

Day 2

1 8 students took part in exam that contains 8 questions. If it is known that each question was solved by at least 5 students, prove that we can always find 2 students such that each of questions was solved by at least one of them.

2 Let AD be height of triangle $\triangle ABC$ and R circumradius. Denote by E and F feet of perpendiculars from point D to sides AB and AC.

If $AD = R\sqrt{2}$, prove that circumcenter of triangle $\triangle ABC$ lies on line EF.

3 Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(f(x) + y) = f(x^2 - y) + 4f(x)y$$

for all $x, y \in \mathbb{R}$.