Bosnia Herzegovina Team Selection Test 2010

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by gobathegreat

Day 1 May 15th

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1 $a)$ Let p and q be distinct prime numbers such that $p+q^2$ divides p^2+q. Prove that $p+q^2$ divides $pq-1$. $b)$ Find all prime numbers p such that $p+121$ divides p^2+11.

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2   Let AB and FD be chords in circle, which does not intersect and P point on arc AB which does not contain chord FD. Lines PF and PD intersect chord AB in Q and R. Prove that $\frac{AQ*RB}{QR}$ is constant, while point P moves along the ray AB.

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3   Find all functions $f : \mathbb{Z} \mapsto \mathbb{Z}$ such that following conditions holds: $a)$ $f(n) \cdot f(-n) = f(n^2)$ for all $n \in \mathbb{Z}$ $b)$ $f(m + n) = f(m) + f(n) + 2mn$ for all $m, n \in \mathbb{Z}$

Day 2 May 16th

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4   Convex quadrilateral is divided by diagonals into four triangles with congruent inscribed circles. Prove that this quadrilateral is rhombus.

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5   Let a,b and c be sides of a triangle such that $a+b+c \le 2$. Prove that $-3 < \frac{a^3}{b} + \frac{b^3}{c} + \frac{c^3}{a} - \frac{a^3}{c} - \frac{b^3}{a} - \frac{c^3}{b} < 3$

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6   Prove that total number of ones which is showed in all nonrestricted partitions of natural number n is equal to sum of numbers of distinct elements in that partitions.