Bosnia Herzegovina Team Selection Test 2011

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Day 1

1 In triangle ABC it holds $|BC| = \frac{1}{2}(|AB| + |AC|)$. Let M and N be midpoints of AB and AC, and let I be the incenter of ABC. Prove that A, M, I, N are concyclic.

2 On semicircle, with diameter $|AB| = d$, are given points C and D such that: $|BC| = |CD| = a$ and $|DA| = b$ where a, b, d are different positive integers. Find minimum possible value of d

3 Numbers $1, 2, ..., 2n$ are partitioned into two sequences $a_1 < a_2 < ... < a_n$ and $b_1 > b_2 > ... > b_n$. Prove that number

$$W = |a_1 - b_1| + |a_2 - b_2| + ... + |a_n - b_n|$$

is a perfect square.

Day 2

1 Find maximum value of number a such that for any arrangement of numbers $1, 2, \ldots, 10$ on a circle, we can find three consecutive numbers such their sum bigger or equal than a.

2 Let a, b, c be positive reals such that $a + b + c = 1$. Prove that the inequality

$$a\sqrt[3]{1 + b - c} + b\sqrt[3]{1 + c - a} + c\sqrt[3]{1 + a - b} \leq 1$$

holds.

3 In quadrilateral $ABCD$ sides AD and BC aren't parallel. Diagonals AC and BD intersect in E. F and G are points on sides AB and DC such $\frac{AF}{FB} = \frac{DG}{GC} = \frac{AD}{BC}$ Prove that if E, F, G are collinear then $ABCD$ is cyclic.