Bosnia Herzegovina Team Selection Test 2013

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Day 1

1 Triangle ABC is right angled at C. Lines AM and BN are internal angle bisectors. AM and BN intersect altitude CH at points P and Q respectively.
Prove that the line which passes through the midpoints of segments QN and PM is parallel to AB.

2 The sequence a_n is defined by $a_0 = a_1 = 1$ and $a_{n+1} = 14a_n - a_{n-1} - 4$,for all positive integers n.
Prove that all terms of this sequence are perfect squares.

3 Prove that in the set consisting of $\binom{2n}{n}$ people we can find a group of $n + 1$ people in which everyone knows everyone or noone knows noone.

Day 2

4 Find all primes p, q such that p divides $30q - 1$ and q divides $30p - 1$.

5 Let $x_1, x_2, \dots, x_n$ be nonnegative real numbers of sum equal to 1.
Let $F_n = x_1^2 + x_2^2 + \cdots + x_n^2 - 2(x_1x_2 + x_2x_3 + \cdots + x_nx_1)$.
Find:
a) $\min F_3$;
b) $\min F_4$;
c) $\min F_5$.

6 In triangle ABC, I is the incenter. We have chosen points P, Q, R on segments IA, IB, IC respectively such that $IP \cdot IA = IQ \cdot IB = IR \cdot IC$.
Prove that the points I and O belong to Euler line of triangle PQR where O is circumcenter of ABC.