

Bosnia Herzegovina Team Selection Test 2014

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by gobathegreat

Day 1 May 10th

1 Let k be the circle and A and B points on circle which are not diametrically opposite. On minor arc AB lies point arbitrary point C . Let D, E and F be foots of perpendiculars from C on chord AB and tangents of circle k in points A and B . Prove that $CD = \sqrt{CE \cdot CF}$

2 Let a, b and c be distinct real numbers. a) Determine value of $\frac{1+ab}{a-b} \cdot \frac{1+bc}{b-c} + \frac{1+bc}{b-c} \cdot \frac{1+ca}{c-a} + \frac{1+ca}{c-a} \cdot \frac{1+ab}{a-b}$

b) Determine value of $\frac{1-ab}{a-b} \cdot \frac{1-bc}{b-c} + \frac{1-bc}{b-c} \cdot \frac{1-ca}{c-a} + \frac{1-ca}{c-a} \cdot \frac{1-ab}{a-b}$

c) Prove the following inequality $\frac{1+a^2b^2}{(a-b)^2} + \frac{1+b^2c^2}{(b-c)^2} + \frac{1+c^2a^2}{(c-a)^2} \geq \frac{3}{2}$

When does equality holds?

3 Find all nonnegative integer numbers such that $7^x - 2 \cdot 5^y = -1$

Day 2 May 11th

1 Sequence a_n is defined by $a_1 = \frac{1}{2}$, $a_m = \frac{a_{m-1}}{2m \cdot a_{m-1} + 1}$ for $m > 1$. Determine value of $a_1 + a_2 + \dots + a_k$ in terms of k , where k is positive integer.

2 It is given regular n -sided polygon, $n \geq 6$. How many triangles they are inside the polygon such that all of their sides are formed by diagonals of polygon and their vertices are vertices of polygon?

3 Let D and E be foots of altitudes from A and B of triangle ABC , F be intersection point of angle bisector from C with side AB , and O, I and H be circumcenter, center of inscribed circle and orthocenter of triangle ABC , respectively. If $\frac{CF}{AD} + \frac{CF}{BE} = 2$, prove that $OI = IH$.