

China Girls Math Olympiad 2004www.artofproblemsolving.com/community/c3669

by April

Day 1

1 We say a positive integer n is *good* if there exists a permutation a_1, a_2, \dots, a_n of $1, 2, \dots, n$ such that $k + a_k$ is perfect square for all $1 \leq k \leq n$. Determine all the good numbers in the set $\{11, 13, 15, 17, 19\}$.

2 Let a, b, c be positive reals. Find the smallest value of

$$\frac{a+3c}{a+2b+c} + \frac{4b}{a+b+2c} - \frac{8c}{a+b+3c}.$$

3 Let ABC be an obtuse inscribed in a circle of radius 1. Prove that $\triangle ABC$ can be covered by an isosceles right-angled triangle with hypotenuse of length $\sqrt{2} + 1$.

4 A deck of 32 cards has 2 different jokers each of which is numbered 0. There are 10 red cards numbered 1 through 10 and similarly for blue and green cards. One chooses a number of cards from the deck. If a card in hand is numbered k , then the value of the card is 2^k , and the value of the hand is sum of the values of the cards in hand. Determine the number of hands having the value 2004.

Day 2

5 Let u, v, w be positive real numbers such that $u\sqrt{vw} + v\sqrt{wu} + w\sqrt{uv} \geq 1$. Find the smallest value of $u + v + w$.

6 Given an acute triangle ABC with O as its circumcenter. Line AO intersects BC at D . Points E, F are on AB, AC respectively such that A, E, D, F are concyclic. Prove that the length of the projection of line segment EF on side BC does not depend on the positions of E and F .

7 Let p and q be two coprime positive integers, and n be a non-negative integer. Determine the number of integers that can be written in the form $ip + jq$, where i and j are non-negative integers with $i + j \leq n$.

8 When the unit squares at the four corners are removed from a three by three squares, the resulting shape is called a cross. What is the maximum number of non-overlapping crosses placed within the boundary of a 10×11 chessboard? (Each cross covers exactly five unit squares on the board.)