Mikls Schweitzer 1966

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by ehsan2004

1 Show that a segment of length h can go through or be tangent to at most $2\lfloor h/\sqrt{2}\rfloor + 2$ nonoverlapping unit spheres.

L.Fejes-Toth, A. Heppes

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2 Characterize those configurations of n coplanar straight lines for which the sum of angles between all pairs of lines is maximum.

L.Fejes-Toth, A. Heppes

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3 Let $f(n)$ denote the maximum possible number of right triangles determined by n coplanar points. Show that

$$\lim_{n\to\infty} \frac{f(n)}{n^2} = \infty \text{ and } \lim_{n\to\infty} \frac{f(n)}{n^3} = 0.$$

P. Erdos

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4 Let I be an ideal of the ring $\mathbb{Z}\left[x\right]$ of all polynomials with integer coefficients such that

a) the elements of I do not have a common divisor of degree greater than 0, and

b) I contains of a polynomial with constant term 1.

Prove that I contains the polynomial $1 + x + x^2 + ... + x^{r-1}$ for some natural number r.

Gy. Szekeres

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5 A "letter T" erected at point A of the x-axis in the xy-plane is the union of a segment AB in the upper half-plane perpendicular to the x-axis and a segment CD containing B in its interior and parallel to the x-axis. Show that it is impossible to erect a letter T at every point of the x-axis so that the union of those erected at rational points is disjoint from the union of those erected at irrational points.

A.Csaszar

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6 A sentence of the following type if often heard in Hungarian weather reports: "Last night's minimum temperatures took all values between -3 degrees and $+5$ degrees." Show that it would suffice to say, "Both -3 degrees and $+5$ degrees occurred among last night's minimum temperatures." (Assume that temperature as a two-variable function of place and time is continuous.)

A.Csaszar

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7 Does there exist a function $f(x,y)$ of two real variables that takes natural numbers as its values and for which $f(x,y)=f(y,z)$ implies $x=y=z$?

A. Hajnal

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8 Prove that in Euclidean ring R the quotient and remainder are always uniquely determined if and only if R is a polynomial ring over some field and the value of the norm is a strictly monotone function of the degree of the polynomial. (To be precise, there are two trivial cases: R can also be a field or the null ring.)

E. Fried

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9 If $\sum_{m=-\infty}^{+\infty} |a_m| < \infty$, then what can be said about the following expression?

$$\lim_{n\to\infty} \frac{1}{2n+1} \sum_{m=-\infty}^{+\infty} |a_{m-n}+a_{m-n+1}+...+a_{m+n}|.$$

P. Turan

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10 For a real number x in the interval $(0,1)$ with decimal representation

$$0.a_1(x)a_2(x)...a_n(x)...,$$

denote by $n(x)$ the smallest nonnegative integer such that

$$\overline{a_{n(x)+1}a_{n(x)+2}a_{n(x)+3}a_{n(x)+4}} = 1966.$$

Determine $\int_0^1 n(x)dx$. ($\overline{abcd}$ denotes the decimal number with digits a,b,c,d.)

A. Renyi
