Mikls Schweitzer 1973

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by ehsan2004

1 We say that the rank of a group G is at most r if every subgroup of G can be generated by at most r elements. Prove
that here exists an integer s such that for every finite group G of rank 2 the commutator series of G has length less than s.

J. Erdos

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2 Let R be an Artinian ring with unity. Suppose that every idempotent element of R commutes with every element of R whose square is 0. Suppose R is the sum of the ideals A and B. Prove that $AB=BA$.

A. Kertesz

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3 Find a constant $c>1$ with the property that, for arbitrary positive integers n and k such that $n>c^k$, the number of distinct prime factors of $\binom{n}{k}$ is at least k.

P. Erdos

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4 Let $f(n)$ be that largest integer k such that n^k divides $n!$, and let $F(n)=\max_{2\leq m\leq n} f(m)$. Show that

$$\lim_{n\to\infty}\frac{F(n)\log n}{n\log\log n}=1.$$

P. Erdos

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5 Verify that for every $x>0$,

$$\frac{\Gamma'(x+1)}{\Gamma(x+1)}>\log x.$$

P. Medgyessy

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6 If f is a nonnegative, continuous, concave function on the closed interval $[0,1]$ such that $f(0)=1$, then

$$\int_0^1 xf(x)dx\leq\frac{2}{3}\left[\int_0^1 f(x)dx\right]^2.$$

Z. Daroczy

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7 Let us connect consecutive vertices of a regular heptagon inscribed in a unit circle by connected subsets (of the plane of the circle) of diameter less than 1. Show that every continuum (in the plane of the circle) of diameter greater than 4, containing the center of the circle, intersects one of these connected sets.

M. Bognar

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8 What is the radius of the largest disc that can be covered by a finite number of closed discs of radius 1 in such a way that each disc intersects at most three others?

L. Fejes-Toth

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9 Determine the value of

$$\sup_{1 \leq \xi \leq 2} [\log E\xi - E\log \xi],$$

where ξ is a random variable and E denotes expectation.

Z. Daroczy

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10 Find the limit distribution of the sequence η_n of random variables with distribution

$$P\left(\eta_n = \arccos(\cos^2 \frac{(2j-1)\pi}{2n})\right) = \frac{1}{n} \; (j=1,2,...,n) \; .$$

($\arccos(.)$ denotes the main value.)

B. Gyires
