Mikls Schweitzer 1976

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by ehsan2004

1 Assume that R, a recursive, binary relation on $\mathbb{N}$ (the set of natural numbers), orders $\mathbb{N}$ into type ω. Show that if $f(n)$ is the nth element of this order, then f is not necessarily recursive.

L. Posa

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2 Let G be an infinite graph such that for any countably infinite vertex set A there is a vertex p, not in A, joined to infinitely many elements of A. Show that G has a countably infinite vertex set A such that G contains uncountably infinitely many vertices p joined to infinitely many elements of A.

P. Erdos, A. Hajnal

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3 Let H denote the set of those natural numbers for which $\tau(n)$ divides n, where $\tau(n)$ is the number of divisors of n. Show that

a) $n! \in H$ for all sufficiently large n,

b)H has density 0.

P. Erdos

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4 Let $\mathbb{Z}$ be the ring of rational integers. Construct an integral domain I satisfying the following conditions:

a)$\mathbb{Z} \subsetneqq I$;

b) no element of $I - \mathbb{Z}$ (only in I) is algebraic over $\mathbb{Z}$ (that is, not a root of a polynomial with coefficients in $\mathbb{Z}$);

c) I only has trivial endomorphisms.

E. Fried

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5 Let $S_\nu = \sum_{j=1}^{n} b_j z_j^\nu \ (\nu = 0, \pm 1, \pm 2, ...)$, where the b_j are arbitrary and the z_j are nonzero complex numbers . Prove that

$$|S_0| \leq n \max_{0<|\nu|\leq n} |S_\nu|.$$

G. Halasz

6 Let $0 \leq c \leq 1$, and let η denote the order type of the set of rational numbers. Assume that with every rational number r we associate a Lebesgue-measurable subset H_r of measure c of the interval $[0,1]$. Prove the existence of a Lebesgue-measurable set $H \subset [0,1]$ of measure c such that for every $x \in H$ the set

$$\{r : \; x \in H_r \}$$

contains a subset of type η.

M. Laczkovich

7 Let $f_1, f_2, \dots, f_n$ be regular functions on a domain of the complex plane, linearly independent over the complex field. Prove that the functions $f_i \overline{f}_k, \; 1 \leq i, k \leq n$, are also linearly independent.

L. Lempert

8 Prove that the set of all linearly combinations (with real coefficients) of the system of polynomials $\{x^n + x^{n^2}\}_{n=0}^{\infty}$ is dense in $C[0,1]$.

J. Szabados

9 Let D be a convex subset of the n-dimensional space, and suppose that D' is obtained from D by applying a positive central dilatation and then a translation. Suppose also that the sum of the volumes of D and D' is 1, and $D \cap D' \neq \emptyset$. Determine the supremum of the volume of the convex hull of $D \cup D'$ taken for all such pairs of sets D, D'.

L. Fejes-Toth, E. Makai

10 Suppose that τ is a metrizable topology on a set X of cardinality less than or equal to continuum. Prove that there exists a separable and metrizable topology on X that is coarser that τ.

L. Juhasz

11 Let $\xi_1, \xi_2, ...$ be independent, identically distributed random variables with distribution

$$P(\xi_1 = -1) = P(\xi_1 = 1) = \frac{1}{2}.$$

Write $S_n = \xi_1 + \xi_2 + ... + \xi_n \;(n = 1, 2, ...), \;\; S_0 = 0$, and

$$T_n = \frac{1}{\sqrt{n}} \max_{0 \leq k \leq n} S_k.$$

Prove that $\liminf_{n \to \infty} (\log n) T_n = 0$ with probability one.

P. Revesz