International Zhautykov Olympiad 2010

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Day 1

1 Find all primes p,q such that $p^3-q^7=p-q$.

2 In a cyclic quadrilateral $ABCD$ with $AB=AD$ points M,N lie on the sides BC and CD respectively so that $MN=BM+DN$. Lines AM and AN meet the circumcircle of $ABCD$ again at points P and Q respectively. Prove that the orthocenter of the triangle APQ lies on the segment MN .

3 A rectangle formed by the lines of checkered paper is divided into figures of three kinds: isosceles right triangles (1) with base of two units, squares (2) with unit side, and parallelograms (3) formed by two sides and two diagonals of unit squares (figures may be oriented in any way). Prove that the number of figures of the third kind is even.

`http://up.iranblog.com/Files7/dda310bab8b6455f90ce.jpg`

Day 2

1 Positive integers $1,2,...,n$ are written on blackboard ($n>2$). Every minute two numbers are erased and the least prime divisor of their sum is written. In the end only the number 97 remains. Find the least n for which it is possible.

2 In every vertex of a regular n -gon exactly one chip is placed. At each $step$ one can exchange any two neighbouring chips. Find the least number of steps necessary to reach the arrangement where every chip is moved by $[\frac{n}{2}]$ positions clockwise from its initial position.

3 Let ABC arbitrary triangle ($AB\neq BC\neq AC\neq AB$) And O,I,H it's circum-center, incenter and ortocenter (point of intersection altitudes). Prove, that
1) $\angle OIH>90^0$(2 points)
2)$\angle OIH>135^0$(7 points)

balls for 1) and 2) not additive.