Federal Competition For Advanced Students, Part 2 2002

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Day 1

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1 Consider all possible rectangles that can be drawn on a 8×8 chessboard, covering only whole cells. Calculate the sum of their areas.

What formula is obtained if 8×8 is replaced with $a \times b$, where a, b are positive integers?

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2 Let b be a positive integer. Find all 2002tuples $(a_1, a_2, \ldots, a_{2002})$, of natural numbers such that

$$\sum_{j=1}^{2002} a_j^{a_j} = 2002b^b.$$

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3 Let $ABCD$ and $AEFG$ be two similar cyclic quadrilaterals (with the vertices denoted counterclockwise). Their circumcircles intersect again at point P. Prove that P lies on line BE.

Day 2

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1 Find all polynomials $P(x)$ of the smallest possible degree with the following properties:

(i) The leading coefficient is 200;
(ii) The coefficient at the smallest non-vanishing power is 2;
(iii) The sum of all the coefficients is 4;
(iv) $P(-1) = 0, P(2) = 6, P(3) = 8$.

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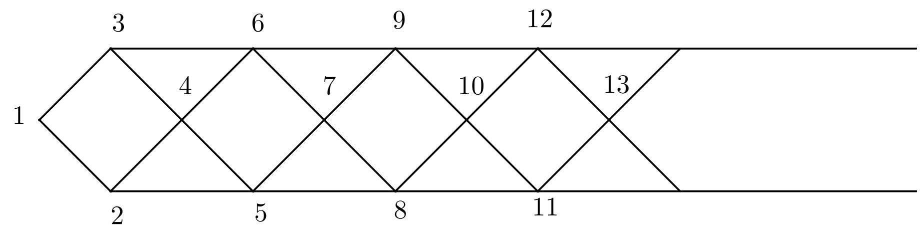
2 In the net drawn below, in how many ways can one reach the point $3n + 1$ starting from the point 1 so that the labels of the points on the way increase?



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3 Let H be the orthocenter of an acute-angled triangle ABC. Show that the triangles ABH, BCH and CAH have the same perimeter if and only if the triangle ABC is equilateral.