Federal Competition For Advanced Students, Part 2 2005

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Day 1

1	Find all triples (a,b,c) of natural numbers, such that $LCM(a,b,c)=a+b+c$
2	Prove that for all positive reals a,b,c,d, we have $\frac{a+b+c+d}{abcd} \le \frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}+\frac{1}{d^3}$
3	Triangle DEF is acute. Circle c_1 is drawn with DF as its diameter and circle c_2 is drawn with DE as its diameter. Points Y and Z are on DF and DE respectively so that EY and FZ are altitudes of triangle DEF . EY intersects c_1 at P, and FZ intersects c_2 at Q. EY extended intersects c_1 at R, and FZ extended intersects c_2 at S. Prove that P, Q, R, and S are concyclic points.

Day 2

1	The function $f:(0,...2005)\rightarrow N$ has the properties that $f(2x+1)=f(2x)$, $f(3x+1)=f(3x)$ and $f(5x+1)=f(5x)$ with $x\in(0,1,2,...,2005)$. How many different values can the function assume?
2	Find all real a,b,c,d,e,f that satisfy the system $4a=(b+c+d+e)^4$ $4b=(c+d+e+f)^4$ $4c=(d+e+f+a)^4$ $4d=(e+f+a+b)^4$ $4e=(f+a+b+c)^4$ $4f=(a+b+c+d)^4$
3	Let Q be a point inside a cube. Prove that there are infinitely many lines l so that $AQ=BQ$ where A and B are the two points of intersection of l and the surface of the cube.