Regional Competition For Advanced Students 1970

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by Binomial-theorem

1 Let x, y, z be positive real numbers such that $x+y+z=1$ Prove that always $\left(1+\frac{1}{x}\right)\times\left(1+\frac{1}{y}\right)\times\left(1+\frac{1}{z}\right)\geq 64$
When does equality hold?

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2 In the plane seven different points $P_1, P_2, P_3, P_4, Q_1, Q_2, Q_3$ are given. The points P_1, P_2, P_3, P_4 are on the straight line p, the points Q_1, Q_2, Q_3 are not on p. By each of the three points Q_1, Q_2, Q_3 the perpendiculars are drawn on the straight lines connecting points different of them. Prove that the maximum's number of the possibles intersections of all perpendiculars is to 286, if the points Q_1, Q_2, Q_3 are taken in account as intersections.

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3 E_1 and E_2 are parallel planes and their distance is p.
(a) How long is the seitenkante of the regular octahedron such that a side lies in E_1 and another in E_2?
(b) E is a plane between E_1 and E_2, parallel to E_1 and E_2, so that its distances from E_1 and E_2 are in ratio $1:2$
Draw the intersection figure of E and the octahedron for $P=4\sqrt{\frac{3}{2}}$ cm and justifies, why the that figure must look in such a way

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4 Find all real solutions of the following set of equations:

$$72x^3+4xy^2=11y^3$$

$$27x^5-45x^4y-10x^2y^3=\frac{-143}{32}y^5$$
