Regional Competition For Advanced Students 2004

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by valerie

1 Determine all integers a and b, so that $(a^3+b)(a+b^3)=(a+b)^4$

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2 Solve the following equation for real numbers: $\sqrt{4-x\sqrt{4-(x-2)\sqrt{1+(x-5)(x-7)}}}=\frac{5x-6-x^2}{2}$

(all square roots are non negative)

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3 Given is a convex quadrilateral $ABCD$ with $\angle ADC=\angle BCD>90^\circ$.
Let E be the point of intersection of the line AC with the parallel line to AD through B and F be the point of intersection of the line BD with the parallel line to BC through A. Show that EF is parallel to CD

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4 The sequence $<x_n>$ is defined through: $x_{n+1}=\left(\frac{n}{2004}+\frac{1}{n}\right)x_n^2-\frac{n^3}{2004}+1$ for $n>0$
Let x_1 be a non-negative integer smaller than 204 so that all members of the sequence are non-negative integers.
Show that there exist infinitely many prime numbers in this sequence.

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