Regional Competition For Advanced Students 2005
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by FelixD

1 Show for all integers $n \geq 2005$ the following chaine of inequalities: $(n+830)^{2005} < n(n+1)\dots(n+2004) < (n+1002)^{2005}$

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2 Construct the semicircle h with the diameter AB and the midpoint M. Now construct the semicircle k with the diameter MB on the same side as h. Let X and Y be points on k, such that the arc BX is $\frac{3}{2}$ times the arc BY. The line MY intersects the line BX in D and the semicircle h in C.
Show that Y ist he midpoint of CD.

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3 For which values of k and d has the system $x^3+y^3=2$ and $y=kx+d$ no real solutions (x,y)?

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4 Prove: if an infinte arithmetic sequence ($a_n=a_0+nd$) of positive real numbers contains two different powers of an integer $a>1$, then the sequence contains an infinite geometric sequence ($b_n=b_0q^n$) of real numbers.
