Regional Competition For Advanced Students 2007

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by FelixD

1 Let $0 < x_0, x_1, \dots , x_{669} < 1$ be pairwise distinct real numbers. Show that there exists a pair (x_i, x_j) with $0 < x_ix_j(x_j - x_i) < \frac{1}{2007}$

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2 Find all tuples $(x_1, x_2, x_3, x_4, x_5)$ of positive integers with $x_1 > x_2 > x_3 > x_4 > x_5 > 0$ and $\left\lfloor \frac{x_1+x_2}{3} \right\rfloor^2 + \left\lfloor \frac{x_2+x_3}{3} \right\rfloor^2 + \left\lfloor \frac{x_3+x_4}{3} \right\rfloor^2 + \left\lfloor \frac{x_4+x_5}{3} \right\rfloor^2 = 38.$

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3 Let a be a positive real number and n a non-negative integer. Determine $S - T$, where $S = \sum_{k=-2n}^{2n+1} \frac{(k-1)^2}{a^{| \lfloor \frac{k}{2} \rfloor |}}$ and $T = \sum_{k=-2n}^{2n+1} \frac{k^2}{a^{| \lfloor \frac{k}{2} \rfloor |}}$

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4 Let M be the intersection of the diagonals of a convex quadrilateral $ABCD$. Determine all such quadrilaterals for which there exists a line g that passes through M and intersects the side AB in P and the side CD in Q, such that the four triangles APM, BPM, CQM, DQM are similar.
