Regional Competition For Advanced Students 2008

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by valerie

1 Show: For all real numbers a, b, c with $0 < a, b, c < 1$ is:

$$\sqrt{a^2bc+ab^2c+abc^2}+\sqrt{(1-a)^2(1-b)(1-c)+(1-a)(1-b)^2(1-c)+(1-a)(1-b)(1-c)^2}<\sqrt{3}.$$

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2 For a real number x is $[x]$ the next smaller integer to x, that is the integer g with $g \leqq < g+1$, and $\{x\} = x - [x]$ is the decimal part of x.
Determine all triples (a, b, c) of real numbers, which fulfil the following system of equations:

$$\{a\}+[b]+\{c\}=2,9$$

$$\{b\}+[c]+\{a\}=5,3$$

$$\{c\}+[a]+\{b\}=4,0$$

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3 Given is an acute angled triangle ABC. Determine all points P inside the triangle with

$$1 \leq \frac{\angle APB}{\angle ACB}, \frac{\angle BPC}{\angle BAC}, \frac{\angle CPA}{\angle CBA} \leq 2$$

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4 For every positive integer n let

$$a_n = \sum_{k=n}^{2n} \frac{(2k+1)^n}{k}$$

Show that there exists no n, for which a_n is a non-negative integer.
