Federal Competition For Advanced Students, Part 1 2002

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by Amir Hossein

1 Determine all integers a and b such that

$$(19a+b)^{18}+(a+b)^{18}+(a+19b)^{18}$$

is a perfect square.

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2 Find the greatest real number C such that, for all real numbers x and $y \neq x$ with $xy=2$ it holds that

$$\frac{((x+y)^2-6)((x-y)^2+8)}{(x-y)^2} \geq C.$$

When does equality occur?

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3 Let $f(x)=\frac{9^x}{9^x+3}$. Compute $\sum_k f\left(\frac{k}{2002}\right)$, where k goes over all integers k between 0 and 2002 which are coprime to 2002.

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4 Let A, C, P be three distinct points in the plane. Construct all parallelograms $ABCD$ such that point P lies on the bisector of angle DAB and $\angle APD = 90^\circ$.
