Federal Competition For Advanced Students, Part 1 2003

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1 Find all triples of prime numbers (p, q, r) such that $p^q + p^r$ is a perfect square.

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2 Find the greatest and smallest value of $f(x, y) = y - 2x$, if x, y are distinct non-negative real numbers with $\frac{x^2+y^2}{x+y} \leq 4$.

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3 Given a positive real number t, find the number of real solutions a, b, c, d of the system

$$a(1-b^2) = b(1-c^2) = c(1-d^2) = d(1-a^2) = t.$$

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4 In a parallelogram $ABCD$, points E and F are the midpoints of AB and BC, respectively, and P is the intersection of EC and FD. Prove that the segments AP, BP, CP and DP divide the parallelogram into four triangles whose areas are in the ratio $1 : 2 : 3 : 4$.
