**Federal Competition For Advanced Students, Part 1 2004**

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**1** Find all quadruples $(a,b,c,d)$ of real numbers such that

$$a+bcd=b+cda=c+dab=d+abc.$$

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**2** A convex hexagon $ABCDEF$ with $AB=BC=a, CD=DE=b, EF=FA=c$ is inscribed in a circle. Show that this hexagon has three (pairwise disjoint) pairs of mutually perpendicular diagonals.

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**3** For natural numbers $a,b$, define $Z(a,b)=\frac{(3a)!\cdot(4b)!}{a!^4\cdot b!^3}$.

**(a)** Prove that $Z(a,b)$ is an integer for $a\leq b$.

**(b)** Prove that for each natural number $b$ there are infinitely many natural numbers a such that $Z(a,b)$ is not an integer.

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**4** Each of the $2N=2004$ real numbers $x_1,x_2,\ldots,x_{2004}$ equals either $\sqrt{2}-1$ or $\sqrt{2}+1$. Can the sum $\sum_{k=1}^N x_{2k-1}x_2k$ take the value $2004$? Which integral values can this sum take?

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