Federal Competition For Advanced Students, Part 1 2005

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1 Prove that there are infinitely many multiples of 2005 that contain all the digits 0, 1, 2,...,9 an equal number of times.

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2 For how many integers a with $|a| \leq 2005$, does the system $x^2 = y + a$ $y^2 = x + a$ have integer solutions?

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3 For 3 real numbers a, b, c let $s_n = a^n + b^n + c^n$.
It is known that $s_1 = 2$, $s_2 = 6$ and $s_3 = 14$.
Prove that for all natural numbers $n > 1$, we have $|s_n^2 - s_{n-1}s_{n+1}| = 8$

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4 We're given two congruent, equilateral triangles ABC and PQR with parallel sides, but one has one vertex pointing up and the other one has the vertex pointing down. One is placed above the other so that the area of intersection is a hexagon $A_1A_2A_3A_4A_5A_6$ (labelled counterclockwise). Prove that A_1A_4, A_2A_5 and A_3A_6 are concurrent.
