Federal Competition For Advanced Students, Part 1 2006

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1 Let n be a non-negative integer, which ends written in decimal notation on exactly k zeros, but which is bigger than 10^k.
For a n is only $k = k(n) \geq 2$ known. In how many different ways (as a function of $k = k(n) \geq 2$) can n be written as difference of two squares of non-negative integers at least?

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2 Show that the sequence $a_n = \frac{(n+1)^n n^{2-n}}{7n^2+1}$ is strictly monotonically increasing, where $n = 0, 1, 2, \ldots$.

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3 In the triangle ABC let D and E be the boundary points of the incircle with the sides BC and AC. Show that if $AD = BE$ holds, then the triangle is isoceles.

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4 Given is the function $f = \lfloor x^2 \rfloor + \{x\}$ for all positive reals x. ($\lfloor x \rfloor$ denotes the largest integer less than or equal x and $\{x\} = x - \lfloor x \rfloor$.)
Show that there exists an arithmetic sequence of different positive rational numbers, which all have the denominator 3, if they are a reduced fraction, and dont lie in the range of the function f.
