Federal Competition For Advanced Students, Part 1 2007

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1 In a quadratic table with 2007 rows and 2007 columns is an odd number written in each field. For $1 \leq i \leq 2007$ is Z_i the sum of the numbers in the i-th row and for $1 \leq j \leq 2007$ is S_j the sum of the numbers in the j-th column. A is the product of all Z_i and B the product of all S_j. Show that $A+B \neq 0$

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2 For every positive integer n determine the highest value $C(n)$, such that for every n-tuple $(a_1, a_2, \ldots, a_n)$ of pairwise distinct integers
$(n+1)\sum_{j=1}^{n} a_j^2 - \left(\sum_{j=1}^{n} a_j\right)^2 \geq C(n)$

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3 Let $M(n) = \{-1, -2, \ldots, -n\}$. For every non-empty subset of $M(n)$ we consider the product of its elements. How big is the sum over all these products?

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4 Let $n > 4$ be a non-negative integer. Given is the in a circle inscribed convex n-gon $A_0A_1A_2\ldots A_{n-1}A_n$ $(A_n = A_0)$ where the side $A_{i-1}A_i = i$ (for $1 \leq i \leq n$). Moreover, let ϕ_i be the angle between the line A_iA_{i+1} and the tangent to the circle in the point A_i (where the angle ϕ_i is less than or equal 90^o, i.e. ϕ_i is always the smaller angle of the two angles between the two lines). Determine the sum $\Phi = \sum_{i=0}^{n-1} \phi_i$
of these n angles.
