Federal Competition For Advanced Students, Part 1 2013

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by Martin N.

1 Show that if for non-negative integers m, n, N, k the equation

$$(n^2+1)^{2^k}\cdot(44n^3+11n^2+10n+2)=N^m$$

holds, then $m=1$.

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2 Solve the following system of equations in rational numbers:

$$(x^2+1)^3=y+1,(y^2+1)^3=z+1,(z^2+1)^3=x+1.$$

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3 Arrange the positive integers into two lines as follows:

1	3		6		11		19		32		53 ...		
		2		4 5		7 8 9 10		12 13 14 15 16 17 18		20 to 31		33 to 52	...

We start with writing 1 in the upper line, 2 in the lower line and 3 again in the upper line. Afterwards, we alternately write one single integer in the upper line and a block of integers in the lower line. The number of consecutive integers in a block is determined by the first number in the previous block.

Let a_1, a_2, a_3, $\ldots$ be the numbers in the upper line. Give an explicit formula for a_n.

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4 Let A, B and C be three points on a line (in this order).

For each circle k through the points B and C, let D be one point of intersection of the perpendicular bisector of BC with the circle k. Further, let E be the second point of intersection of the line AD with k.

Show that for each circle k, the ratio of lengths $\overline{BE}:\overline{CE}$ is the same.
