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1 It is given the sequence defined by

$$\{a_{n+2} = 6a_{n+1} - a_n\}_{n\in\mathbb{Z}_{>0}}, a_1 = 1, a_2 = 7.$$

Find all n such that there exists an integer m for which $a_n = 2m^2 - 1$.

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2 Let S a finite subset of $\mathbb{N}$. For every positive integer i, let A_i the number of partitions of i with all parts in $\mathbb{N} - S$.
Prove that there exists $M \in \mathbb{N}$ such that $A_{i+1} > A_i$ for all $i > M$.
($\mathbb{N}$ is the set of positive integers)

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3 Let Γ a fixed circumference. Find all finite sets S of points in Γ such that:
For each point $P \in \Gamma$, there exists a partition of S in sets A and B
($A \cup B = S$, $A \cap B = \phi$) such that $\sum_{X\in A} PX = \sum_{Y\in B} PY$.

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4 Let Ω and Γ two circumferences such that Ω is in interior of Γ. Let P a point on Γ.
Define points A and B distinct of P on Γ such that PA and PB are tangentes to Ω. Prove that when P
varies on Γ, the line AB is tangent to a fixed circumference.

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5 Let T the set of the infinite sequences of integers. For two given elements in T: $(a_1, a_2, a_3, ...)$ and $(b_1, b_2, b_3, ...)$, define the sum $(a_1, a_2, a_3, ...) + (b_1, b_2, b_3, ...) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, ...)$.
Let $f : T \to \mathbb{Z}$ a function such that:
i) If $x \in T$ has exactly one of your terms equal 1 and all the others equal 0, then $f(x) = 0$.
ii)$f(x + y) = f(x) + f(y)$, for all $x, y \in T$.
Prove that $f(x) = 0$ for all $x \in T$
